Direct and inverse proportion

How to write equations for directly proportional relationships and inversely proportional relationships.

Direct proportion

If two quantities increase or decrease at the same rate, they are said to be **directly proportional**.

This means that if one quantity is **doubled**, the other quantity is **doubled**; if one quantity is **halved**, the other quantity is **halved** etc.

The symbol for proportion is $\propto$.

To write 'y is directly proportional to x' using the proportionality symbol, we would write:
$y \propto x$.

Proportionality can be used to set up an equation; to do this:
1) Write the proportional relationship.
2) Convert this to an equation including a constant of proportionality.
3) Use given information to find the constant of proportionality.
4) Substitute the constant of proportionality into the equation.

Example 1

The value of y is ***directly proportional*** to the value of x.

When $y = 20$, $x = 5$.

Use this information to write an equation relating y and x.

Answer

$$y \propto x$$
$$y = kx$$
$$20 = 5k$$
$$\frac{20}{5} = k$$
$$\text{so } k = 4$$
$$y = 4x$$

Example 2

The value of t is ***directly proportional*** to the value of w^2.

When $t = 1$, $w = 2$.

Use this information to write an equation relating t and w.

Answer

$$t \propto w^2$$
$$t = kw^2$$
$$1 = 4k$$
$$\frac{1}{4} = k$$
$$\text{so } k = 0{\cdot}25$$
$$t = 0{\cdot}25w^2$$

Inverse proportion

If one quantity increases at the same rate as another decreases, the quantities are said to be **inversely proportional**.

This means that if one quantity is **doubled**, the other quantity is **halved**; if one quantity is **multiplied by 3**, the other quantity is **divided by 3** etc.

To write 'y is inversely proportional to x' using the proportionality symbol, we would write:

$$y \propto \frac{1}{x}.$$

We can set up an equation for an inversely proportional relationship using the same steps as we did for a direct proportional relationship (on the left).

Example 3

The value of y is ***inversely proportional*** to the value of x.

When $y = 15$, $x = 5$.

Use this information to write an equation relating y and x.

Answer

$$y \propto \frac{1}{x}$$
$$y = k \times \frac{1}{x} = \frac{k}{x}$$
$$15 = \frac{k}{5}$$
$$15 \times 5 = k$$
$$\text{so } k = 75$$
$$y = \frac{75}{x}$$

Example 4

The value of t is ***inversely proportional*** to the value of w^2.

When $t = 8$, $w = 3$.

Use this information to write an equation relating t and w.

Answer

$$t \propto \frac{1}{w^2}$$
$$t = \frac{k}{w^2}$$
$$8 = \frac{k}{3^2}$$
$$8 = \frac{k}{9}$$
$$8 \times 9 = k$$
$$\text{so } k = 72$$
$$t = \frac{72}{w^2}$$

Check that you can:
- substitute values into an equation
- change the subject of a formula.

REMEMBER!

If two quantities are directly proportional, doing something to one quantity changes the other in the same way.

If two quantities are inversely proportional, doing something to one quantity causes an opposite change to the other quantity.