

## Section 1.1a – Operations with Integers

This booklet belongs to: \_\_\_\_\_ Block: \_\_\_\_\_

### Adding and Subtracting Integers

- They represent all the **countable numbers**, both **positive** and **negative**

$$(\dots - 3, -2, -1, 0, 1, 2, 3, \dots)$$

- A great place to start is to **understand** that **subtraction** can be shown as **adding negatives**, everything **can be written as an addition statement** when we are using integers.

**Example:**  $7 - 4 = 7 + (-4)$  This may seem weird now, but it will come in handy later

If this helps, think of positive and negatives as:

*Positive – good things*

*Negative – bad things*

- When **adding and subtracting think** of adding and taking away good and bad things
- All you need to consider then is **which did you have more of** in the beginning

### Examples:

$6 - 2 = 4$

$5 + (-3) = 2$

$-4 - 8 = -12$

$12 - 14 = -2$

$-7 + 4 = -3$

$-7 + (-2) = -9$

- When we **subtract negatives** don't think 'subtract', but think: **Take Away**

$5 - (-3)$  You have **5 good things** and you **take away 3 bad things**

- Since you **don't have bad things** to begin with **introduce some in equilibrium (zero)**
- Now you can **take away the bad**, but it **leaves the good** you brought.

**Using a Diagram**

5 - (-3)      What do you start with?

5 positives  
+ + + + +

Need to take away negatives.  
So, you'll need some.

+ + +  
- - -

This is zero.

Now you **can take away the negatives**.

What are you left with?

+ + + + + + + +  
8 positives

**Example 1:** Use diagrams to solve the following:  $-4 - (-3)$

**Solution 1:** What do you start with?

$-4 - (-3)$

4 negatives  
- - - -

This situation is easier since **we have what we need to take away**. Just take 3 negatives away.

-  
1 negative

$-4 - (-3) = -1$

**Example 2:** Use diagrams to solve the following:  $5 - (-2)$

**Solution 2:** What do you start with?

$5 - (-2)$

5 positives  
+ + + + +

+ +  
- -

I need negatives to take away.

Now you can **take away the negatives**.

7 positives  
+ + + + + + +

This is 0

What are you left with?

$5 - (-2) = 7$

**Example 3:** Use diagrams to solve the following:  $-6 - 4$

**Solution 3:**

$-6 - 4$

6 negatives  
- - - - -

- - - -  
+ + + +

I need Positives to take away.

Now you **can take away the positives**.

10 negatives  
- - - - -

This is 0

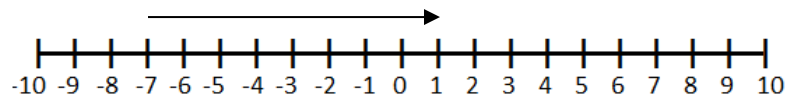
$-6 - (4) = -10$

**Moving on a Number Line (Since everything can be written as an addition statement)**

- If we **add two numbers** together, say:  $m + n$ 
  - If  $n$  is **positive**, we **move to the right**
  - If  $n$  is **negative**, we **move to the left**
  - If  $n$  is **zero**, we **do not move**

**Example 1:** Find  $-7 + 8$

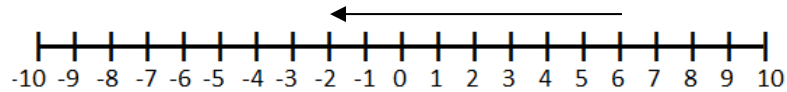
**Solution 1:** Start at  $-7$  and move **8 units to the right**



So:  $-7 + 8 = 1$

**Example 2:** Find  $6 + (-8)$

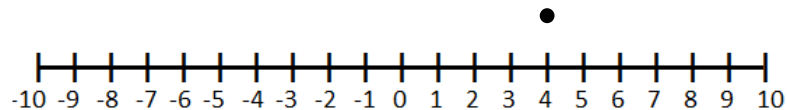
**Solution 2:** Start at 6 and move **8 units to the left**



So:  $6 + (-8) = -2$

**Example 3:** Find  $4 + 0$

**Solution 3:** Start at 4 and **do not move**



So:  $4 + 0 = 4$

**Adding Integers without Diagrams**

- Adding numbers is relatively straight forward
- We have been doing it all of our lives
- When negatives get mixed into the game, people start to get confused
- Let’s try to fix that

***Two Positive Numbers (Good Things)***

Add the Numbers, Answer is Positive (Good things + Good Things = Better Things)       $2 + 3 = 5$

***Two Negative Numbers (Bad Things)***

Add the Numbers, Answer is Negative (Bad things + Bad Things = Worse Things)       $-2 + (-3) = (-5)$

***A Positive and a Negative (Good and Bad Things; Comes down to What we Had More of to Start)***

- i)      **More Positives** to Start, **Answer is Positive**       $5 + (-3) = 2$
- ii)     **More Negatives** to Start, **Answer is** the same number, but **Negative**       $-5 + 3 = -2$

**Subtracting Integers**

- Subtracting gets a little tricky. But it helps to consider:

Subtraction is just the addition (sum) of a negative:       $a - b = a + (-b)$

- It may also help to channel your inner child and **instead of subtract**, think **Take Away**

**Subtraction**

$4 - 7 = -3$       Start with Good things, take away more good things than you had, that’s bad

$-5 - 3 = -8$       Start with Bad things, take away good things, that’s making things worse

$-6 - (-8) = 2$       Start with Bad things, take away more Bad things than you had, makes things

$-4 - (-3) = -1$       Start with Bad things, take away some of them, still a bad day

What we can do though, is we can **re-write the subtraction** equation as an **addition statement**

$4 - 7 = -3 \quad \rightarrow \quad 4 + (-7) = -3$	$-5 - 3 = -8 \quad \rightarrow \quad -5 + (-3) = -8$
$-6 - (-8) = 2 \quad \rightarrow \quad -6 + 8 = 2$	$-4 - (-3) = -1 \quad \rightarrow \quad -4 + 3 = -1$

- Lastly, when dealing with a statement with a positive and negative number, consider this:

Regardless of the order the **difference between the two number is the same** what **changes is the sign**.

**Example:**

$$5 - 2 \quad \rightarrow \quad 5 + (-2) = 3$$

$$2 - 5 \quad \rightarrow \quad -5 + 2 = -3$$

The **difference is still 3**, what changes is the sign. If you **started with more negatives** the **answer is negative**. If you **started with more positives** the **answer is positive**.

What this means is that if you are stuck with challenging numbers, just **subtract the smaller number (independent of sign) from the larger number (independent of sign)** and the answer is the answer. Just **put a negative sign** if the **larger number was the negative one**.

**Example:** Consider  $-10 + 6$

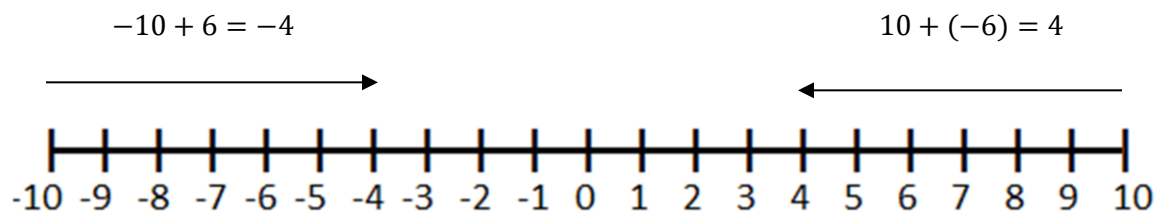
**Solution:** Independent of sign, 10 is bigger than 6.

Since you are adding one negative and one positive, follow the directions mentioned above.

$$10 + (-6) = 4$$

But since you started with  $-10$  the answer has to be negative.

$$-10 + 6 = -4$$



**Example 4:** Find  $5.43 + 3.12$

**Solution 4:** This is straightforward, just add the numbers together, be sure to line up the decimal points.

$$\begin{array}{r} 5.43 \\ + 3.12 \\ \hline 8.55 \end{array} \quad \text{So,} \quad \mathbf{5.43 + 3.12 = 8.55}$$

**Example 5:** Find  $5.43 + (-3.12)$

**Solution 5:** This is straightforward since the larger number is positive, just subtract the numbers as you would traditionally, be sure to line up the decimal points.

$$\begin{array}{r} 5.43 \\ - 3.12 \\ \hline 2.31 \end{array} \quad \text{So,} \quad \mathbf{5.43 + (-3.12) = 2.31}$$

**Example 6:** Find  $-5.43 + 3.12$

**Solution 6:** Since the larger number is negative, just subtract the numbers as you would traditionally, but the answer will be negative. Line up the decimal points.

$$\begin{array}{r} 5.43 \\ - 3.12 \\ \hline 2.31 \end{array} \quad \text{So,} \quad \mathbf{-5.43 + 3.12 = -2.31}$$

**Example 7:** Find  $-5.43 - 3.12$

**Solution 7:**  $-5.43 - 3.12 \rightarrow -5.43 + (-3.12)$ , so add the numbers together, but the answer should be negative. Line up the decimal points.

$$\begin{array}{r} 5.43 \\ + 3.12 \\ \hline 8.55 \end{array} \quad \text{So,} \quad \mathbf{-5.43 - 3.12 = -8.55}$$

**Multiplying and Dividing Integers**

- When **multiplying and dividing** integers, **two wrongs make a right** and **two rights make a right**

$$+ * + = +$$

$$- * - = +$$

**Multiplying/Dividing the Same Signs is always Positive**

$$+ * - = -$$

$$- * + = -$$

**Multiplying/Dividing Opposite Signs is always Negative**

**Examples:**

$$5 \cdot (-4) = -20$$

$$12 \div 3 = 4$$

$$-2 \cdot (-3) = 6$$

$$(-7) \cdot (-4) = 28$$

$$24 \div 3 = 8$$

$$2 \cdot -(-4) = 8$$

$$-18 \div 2 = -9$$

$$5 \cdot (-4) = -20$$

$$15 \div (-5) = -3$$

**Section 1.1a – Practice Problems****EMERGING LEVEL QUESTIONS**

Execute the following operations by displaying diagrams of the situation, what do you start with?

1.  $3 + (-2)$

2.  $(-5) + (-7)$

3.  $3 - (-5)$

4.  $12 - 7$

5.  $-7 - 4$

Add the following Integers without a calculator

6.  $4 + 7$

7.  $4 + (-7)$

8.  $(-4) + (-7)$

9.  $-4 + 7$

10.  $4 + 3 + 6$

11.  $4 + (-3) + 6$



**PROFICIENT LEVEL QUESTIONS**

12.  $10 + 5 + (-12)$

13.  $4 + (-5) + 12$

14.  $-4 + (-5) + 7$

15.  $-7 + 3 + (-5)$

Subtract the following Integers without a calculator

**EMERGING LEVEL QUESTIONS**

16.  $18 - 5$

17.  $-18 - 7$

18.  $-4 - (-7)$

19.  $4 - 7$

**PROFICIENT LEVEL QUESTIONS**

20.  $-13 - 8 - (-4)$

21.  $-15 - 6 - 3$

22.  $-7 - (-4) - (-6)$

23.  $-12 - (-15) - 4$

24.  $14 - (-5) - 9$

25.  $21 - (-7) - 10$

**EXTENDING LEVEL QUESTIONS**

Add and Subtract the following decimal integers without a calculator

26.  $-4.06 + 1.83$

27.  $-5.637 + (-3.71)$

28.  $4.06 - 1.83$

29.  $-5.637 - (-3.711)$

30.  $7.204 - (-1.8)$

31.  $-7.204 + (-1.8)$

**EMERGING LEVEL QUESTIONS**

Multiply and Divide the following integers without a calculator

32.  $-4 \cdot 7$

33.  $-4 \cdot (-7)$

34.  $2 \cdot (-9)$

35.  $-4 \cdot 7$

36.  $4 \cdot 3 \cdot 6$

37.  $4 \cdot (-3) \cdot 6$

38.  $10 \cdot 5 \cdot (-12)$

39.  $4 \cdot (-5) \cdot 12$

40.  $-40 \div (-5)$

41.  $-72 \div 3$

**PROFICIENT LEVEL QUESTIONS**

42.  $-112 \div 2$

43.  $-200 \div 5$

44.  $-70 \div 2 \cdot (-1)$

45.  $28 \div (-4) \cdot (-3)$

46.  $-56 \div (-8) \cdot (-6)$

47.  $720 \div -3 \cdot (-3)$

**EXTENDING LEVEL QUESTIONS**

Transform the written statements into a numerical statement and solve it.

- |   |  |
|---|--|
| <p>48. My mother gave me \$25 dollars to buy food. I decide to order on Uber Eats and the meal cost me \$13, the delivery cost me \$4, and I tipped the driver \$2. How much money do I have left. Was this a good use of my money?</p> | <p>49. My bank account is in overdraft \$42. I get charged an additional \$5 fee, and then pay back \$30. How much do I still owe?</p>   |
| <p>50. In Victoria today is <math>7^{\circ}\text{C}</math> and in Edmonton is <math>15^{\circ}\text{C}</math> below zero. What is the difference in the temperature between the two cities.</p>   | <p>51. The phone I want to buy costs \$1200 outright, I have \$856 saved up, how much am I short?</p>  |
| <p>52. I am in debt \$4200 but I have amazing friends. Three of them said they would split the debt with me, how much do we owe each?</p>   | <p>53. My office has arranged a Holiday party, there are a number of fees to pay though. The booking of the restaurant costs \$600, the transportation costs \$475, and the food costs \$2500. We have \$575 in the staff account to offset the cost. If 300 people are coming, how much do they each owe?</p> |

54. My family trip cost us \$6548. Each member of the family (5 of us) has \$1310 to contribute to the bill. Do we have enough money to pay back the entire cost? By how much are we short or over?
55. 15 people all contribute \$575 over the course of 5 months to a savings fund.
- I) How much do they each contribute per month?
  - II) How much is there in the fund at the end of the 5 months?
  - III) If they need \$9000 in the fund at the end of the 5 months, how much are they over or short?
  - IV) How much would each person need to contribute monthly to make the goal of \$9000?

**Answer Key – Section 1.1a**

1. 1 (See Diagram)	2. -12 (See Diagram)	3. 8 (See Diagram)
4. 5 (See Diagram)	5. -11 (See Diagram)	6. 11
7. -3	8. -11	9. 3
10. 13	11. 7	12. 3
13. 11	14. -2	15. -9
16. 13	17. -25	18. 3
19. -3	20. -17	21. -24
22. 3	23. -1	24. 10
25. 18	26. -2.23	27. -9.347
28. 2.23	29. -1.926	30. 9.004
31. -9.004	32. -28	33. 28
34. -18	35. -28	36. 72
37. -72	38. -600	39. -240
40. 8	41. -24	42. -56
43. -40	44. 35	45. 21
46. -42	47. 720	48. \$6 See Website for Detail.
49. \$17 See Website for Detail.	50. 22°C See Website for Detail.	51. \$ - 344 See Website for Detail.
52. \$ - 1050 Website for Detail.	53. \$10 See Website for Detail.	54. Yes by \$2 Website for Detail.
55. Website for Detail i) \$115 ii) \$8625 iii) Short \$375 iv) \$5 more for a total of \$120 each		